

Spectrum and Decays of Hadronic Atoms

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Abstract

Using non relativistic effective Lagrangian techniques, we analyze the hadronic decay of the $\pi^+\pi^-$ atom and the strong energy-level shift of pionic hydrogen in the ground state. We derive general formulae for the width and level shift, valid at next-to-leading order in isospin breaking. The result is expressed in terms of hadronic threshold amplitudes that include isospin-breaking effects. In order to extract isospin symmetric scattering lengths from the data, we invoke chiral perturbation theory, that allows one to relate the scattering lengths to the threshold amplitudes.

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Recent years have seen a growing interest in the study of hadronic atoms. At CERN, the DIRAC collaboration [1] aims to measure the $\pi^+\pi^-$ atom lifetime to 10% accuracy. This would allow one to determine the difference $a_0 - a_2$ of $\pi\pi$ scattering lengths with 5% precision. This measurement provides a crucial test for the large/small condensate scenario in QCD: should it turn out that the quantity $a_0 - a_2$ is different from the value predicted in standard ChPT [2], one has to conclude [3] that spontaneous chiral symmetry breaking in QCD proceeds differently from the widely accepted picture [4]. In the experiment performed at PSI [5], one has measured the strong energy-level shift and the total decay width of the $1s$ state of pionic hydrogen, as well as the $1s$ shift of pionic deuterium. Using the technique described in Ref. [5], these measurements yield [6] isospin symmetric πN scattering lengths to an accuracy which is unique for hadron physics: $a_{0+}^+ = (1.6 \pm 1.3) \times 10^{-3} M_{\pi^+}^{-1}$ and $a_{0+}^- = (86.8 \pm 1.4) \times 10^{-3} M_{\pi^+}^{-1}$. The scattering length a_{0+}^- may be used as an input in the Goldberger-Miyazawa-Oehme [7] sum rule to determine the πNN coupling constant [5, 6]. A new experiment on pionic hydrogen [8] has recently been approved. It will allow one to measure the decay $A_{\pi^-p} \rightarrow \pi^0 n$ to much higher accuracy and thus enable one, in principle, to determine the πN scattering lengths from data on pionic hydrogen alone. This might vastly reduce the model-dependent uncertainties that come from the analysis of the three-body problem in A_{π^-d} . Finally, the DEAR collaboration [10] at the DAΦNE facility (Frascati) plans to measure the energy level shift and lifetime of the $1s$ state in K^-p and K^-d atoms - with considerably higher precision than in the previous experiment carried out at KEK [11] for K^-p atoms. It is expected [10] that this will result in a precise determination of the $I = 0, 1$ S -wave scattering lengths - although, of course, one will again be faced with the three-body problem already mentioned. It will be a challenge for theorists to extract from this new information on the $\bar{K}N$ amplitude at threshold a more precise value of e.g. the isoscalar kaon-sigma term and of the strangeness content of the nucleon [12].

We now turn to *theoretical* investigations of hadronic atoms. At leading order in isospin breaking, the energy-level shift and the decay width of these atoms can be expressed in terms of the strong hadronic scattering lengths through the well-known formulae by Deser *et al.* [13]. More precisely, these formulae relate the ground state level shift - induced by the

strong interaction - and its partial decay width into neutral hadrons (e.g., $A_{\pi^+\pi^-} \rightarrow \pi^0\pi^0$, $A_{\pi^-p} \rightarrow \pi^0n$) to the corresponding isospin combinations of strong scattering lengths,

$$\Delta E_{\text{str}} \sim \Psi_0^2 \text{Re } a_{cc}, \quad \Gamma_{c0} \sim (\text{phase space}) \times \Psi_0^2 |a_{c0}|^2. \quad (1)$$

Here, Ψ_0 denotes the value of the Coulomb wave function at the origin, and a_{cc} , a_{c0} stand for the relevant isospin combinations of strong scattering lengths. We have used the notation "c" for "charged" (e.g., $\pi^+\pi^-$, π^-p) and "0" for "neutral" (e.g., $\pi^0\pi^0$, π^0n) channels. The accuracy of these leading-order formulae is however not sufficient to fully exploit existing and forthcoming high-precision data on hadronic atoms. Indeed, for that purpose, one has to evaluate isospin-breaking corrections at next-to-leading order. The aim of the present talk is to show how this can be achieved.

Recently, using a non relativistic effective Lagrangian framework, a general expression for the decay width $\Gamma_{A_{2\pi} \rightarrow \pi^0\pi^0}$ of the $1s$ state of the $\pi^+\pi^-$ atom was obtained at next-to-leading order in isospin-breaking [14]. We denote the fine-structure constant α and the quark mass difference squared $(m_d - m_u)^2$ by the common symbol δ . Then, the decay width is written in the following form²,

$$\begin{aligned} \Gamma_{A_{2\pi} \rightarrow \pi^0\pi^0} &= \frac{2}{9} \alpha^3 p^\star \mathcal{A}_{\pi\pi}^2 (1 + K_{\pi\pi}), \quad \mathcal{A}_{\pi\pi} = a_0 - a_2 + O(\delta), \\ K_{\pi\pi} &= \frac{\Delta M_\pi^2}{9M_{\pi^+}^2} (a_0 + 2a_2)^2 - \frac{2\alpha}{3} (\ln \alpha - 1) (2a_0 + a_2) + o(\delta). \end{aligned} \quad (2)$$

Here $p^\star = (M_{\pi^+}^2 - M_{\pi^0}^2 - \frac{1}{4}M_{\pi^+}^2\alpha^2)^{1/2}$, and a_I ($I = 0, 2$) denote the strong $\pi\pi$ scattering lengths in the channel with total isospin I , and the quantity $\mathcal{A}_{\pi\pi}$ is calculated as follows [14]. One calculates the relativistic amplitude for the process $\pi^+\pi^- \rightarrow \pi^0\pi^0$ at $O(\delta)$ in the normalization chosen so that at $O(\delta^0)$ the amplitude at threshold coincides with the difference $a_0 - a_2$ of (dimensionless) S -wave $\pi\pi$ scattering lengths. Due to the presence of virtual photons, the amplitude is multiplied by an overall Coulomb phase that is removed. The real part of the remainder contains terms that diverge like $|\mathbf{p}|^{-1}$ and $\ln 2|\mathbf{p}|/M_{\pi^+}$ at $|\mathbf{p}| \rightarrow 0$ (\mathbf{p}

² We use throughout the Landau symbols $O(x)$ [$o(x)$] for quantities that vanish like x [faster than x] when x tends to zero. Furthermore, it is understood that this holds modulo logarithmic terms, i.e. we write also $O(x)$ for $x \ln x$.

denotes the relative 3-momentum of charged pion pairs). The quantity $\mathcal{A}_{\pi\pi}$ is obtained by subtracting these divergent pieces, and by then evaluating the remainder at $\mathbf{p} = 0$. We shall refer to $\mathcal{A}_{\pi\pi}$ as the physical scattering amplitude at threshold.

A few remarks are in order. As it is seen explicitly from Eq. (2), one can directly extract the value of $\mathcal{A}_{\pi\pi}$ from the measurement of the decay width, because the correction $K_{\pi\pi}$ is very small and the error introduced by it is negligible. We emphasize that in derivation of Eq. (2), chiral expansions have not been used. On the other hand, if one further aims to extract strong scattering lengths from data, one may invoke chiral perturbation theory (ChPT) and to relate the quantities $\mathcal{A}_{\pi\pi}$ and $a_0 - a_2$ order by order in the chiral expansion. This requires the evaluation of isospin-breaking corrections to the scattering amplitude.

The corrections to the hadronic atom characteristics, evaluated in this manner contain, in general, contributions which have not been taken into account so far within the potential scattering approach to the same problem [5, 15]. An obvious candidate for these contributions is the effect coming from the direct quark-photon coupling that is encoded in the so-called "electromagnetic" low-energy constants (LEC's) in ChPT. A second effect is related to the convention-dependent definition of the isospin-symmetric world against which the isospin-breaking corrections are calculated. We adopt the widely used convention that the masses of the isospin multiplets (π^\pm, π^0) and (p, n) in this world coincide with the masses of the charged particles in the real world. This definition induces a contribution to the isospin-breaking corrections in the level shifts and decay widths. We shall display below both corrections explicitly in the case of the π^-p energy-level shift, where these effects emerge already at tree level.

The investigation of the π^-p atom is very similar to the procedure used in the description of the $\pi^+\pi^-$ atom [14]. In the following, we restrict ourselves to the case of the strong energy-level shift of the π^-p atom in the ground state. Because the proton-neutron mass difference contains terms linear in $m_d - m_u$, we count α and $m_d - m_u$ as quantities of the same order, and denote them by the common symbol δ' . [Since this counting is merely a matter of convenience, our previous results on the $\pi^+\pi^-$ atom remain of course unaltered.] Further, for the energy shift of hadronic atoms, one can no longer neglect the electromagnetic

contributions coming from transverse photons as it was done in the case of the width of the $\pi^+\pi^-$ atom. The reason for this can easily be seen from counting powers of α in the energy-level shift. The binding energy of the atom starts at $O(\alpha^2)$ (nonrelativistic value $E_{\text{NR}} = -\frac{1}{2}\mu_c\alpha^2$, where μ_c denotes the reduced mass of π^-p system), and the corresponding QED corrections start at $O(\alpha^4)$. According to Eq. (1), the leading-order strong energy-level shift is $O(\alpha^3)$, while the next-to-leading order corrections start at $O(\alpha^4)$ and should therefore be treated on the same footing as the QED corrections³. QED corrections, however, are not considered here - we focus on the strong energy-level shift alone. For the latter, it is straightforward to obtain a general formula very similar to Eq. (2), that gives the strong energy-level shift including $O(\delta')$ corrections:

$$\Delta E_{\text{str}} = -2\alpha^3\mu_c^2\mathcal{A}_{\pi N}(1 + K_{\pi N}), \quad (3)$$

where $K_{\pi N}$ is a quantity of order δ' (modulo logarithms) and can be expressed in terms of the S -wave πN scattering lengths a_{0+}^+ and a_{0+}^- . Since $K_{\pi N}$ is small, the error introduced by the uncertainty in the determination of a_{0+}^+ , a_{0+}^- is negligible. The major uncertainty in the energy-level shift comes from the quantity $\mathcal{A}_{\pi N}$ whose definition is very similar to that of $\mathcal{A}_{\pi\pi}$. To evaluate this quantity, one has to calculate the relativistic scattering amplitude for the process $\pi^-p \rightarrow \pi^-p$ at $O(\delta')$, subtract all diagrams that are made disconnected by cutting one virtual photon line and remove the Coulomb phase. The real part of the remainder, as for the $\pi^+\pi^-$ case, contains singular pieces that behave like $|\mathbf{p}|^{-1}$ and $\ln|\mathbf{p}|/\mu_c$ that should be again subtracted (\mathbf{p} denotes the relative 3-momentum of the π^-p pair in CM). The rest - evaluated at $\mathbf{p} = 0$ - coincides, by definition, with $\mathcal{A}_{\pi N}$. [The normalization of the relativistic amplitude is chosen so that $\mathcal{A}_{\pi N} = a_{0+}^+ + a_{0+}^- + O(\delta')$.]

Further, to analyze the isospin-breaking corrections to the energy-level shift, we relate the physical scattering amplitude at threshold $\mathcal{A}_{\pi N}$ to the scattering lengths a_{0+}^+ , a_{0+}^- in ChPT.

³ There is one important exception to this rule. Though the vacuum polarization correction starts at $O(\alpha^5)$, it is amplified by a large factor $(\mu_c/m_e)^2$, where m_e denotes the electron mass. Since $\alpha\mu_c/m_e \sim 1$, this contribution is numerically as important as the leading-order strong contribution (see [5]). The graph responsible for this contribution can be, however, easily singled out and the contribution from it merely added to the final result.

At $O(p^2)$ in the chiral expansion, where the amplitude is determined by tree diagrams, this relation is remarkably simple. Constructed on the basis of the effective πN Lagrangian [16, 17, 18], the amplitude contains the pseudovector Born term $\mathcal{A}_{\pi N}^{\text{pv}}$ with physical masses, and a contribution that contains a linear combination of $O(p^2)$ LEC's,

$$\begin{aligned}\mathcal{A}_{\pi N}^{(2)} &= a_{0+}^+ + a_{0+}^- + \epsilon_{\pi N}^{(2)} \\ &= \mathcal{A}_{\pi N}^{\text{pv}} + 2\hat{m}B\kappa_1c_1 + M_{\pi^+}^2(\kappa_2c_2 + \kappa_3c_3) + e^2(\sigma_1f_1 + \sigma_2f_2),\end{aligned}\quad (4)$$

where the quantity B is related to the quark condensate, and where c_i (f_i) are strong (electromagnetic) LEC's from the $O(p^2)$ Lagrangian of ChPT. Furthermore, κ_i and σ_i denote isospin symmetric coefficients whose explicit expressions are not needed here. From Eq. (4), it is straightforward to visualize both mechanisms of isospin-breaking corrections to the hadronic atom observables, not included in potential approaches. The direct quark-photon coupling is encoded in the coupling constants f_i , whereas the effect of the mass tuning in the hadronic amplitude (described above) is due to the term proportional to $2\hat{m}B$. Indeed, at this order in the chiral expansion, one has $2\hat{m}B = M_{\pi^0}^2$. As we express the strong amplitude in terms of charged masses by convention, we write

$$2\hat{m}B = M_{\pi^+}^2 - \Delta_\pi; \quad \Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2, \quad (5)$$

and obtain

$$\epsilon_{\pi N}^{(2)} = -\Delta_\pi \kappa_1c_1 + e^2(\sigma_1f_1 + \sigma_2f_2) + O(\hat{m}\delta') + o(\delta'). \quad (6)$$

Estimates for the energy-level shift on the basis of the expression (6) will be presented elsewhere. Here we note that a simple order-of-magnitude estimate for f_1 shows that f_1 induces an uncertainty in the energy-level shift of roughly the same size as the total correction given in Ref. [5].

To summarize, we have applied a non relativistic effective Lagrangian approach to the study of $\pi^+\pi^-$ and π^-p atoms in the ground state. A general expression for the width $\Gamma_{A_{2\pi} \rightarrow \pi^0\pi^0}$ and for the strong level shift of pionic hydrogen has been obtained at next-to-leading order in isospin breaking. The sources of the isospin-breaking corrections in these

quantities, complementary to ones already considered in the potential scattering theory approach, have been clearly identified.

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